

# Probing Deformed Orbitals with $\vec{A}(\vec{e}, e'N)B$ reactions

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## Abstract

We present results for response functions and asymmetries in the nuclear reactions  $^{37}\vec{Ar}(\vec{e}, e'n)^{36}Ar$  and  $^{37}\vec{K}(\vec{e}, e'p)^{36}Ar$  at quasifree kinematics. We compare PWIA results obtained using deformed HF wave functions with PWIA and DWIA results obtained assuming a spherical mean field. We show that the complex structure of the deformed orbitals can be probed by coincidence measurements with polarized beam and targets.

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# 1 Introduction

The concepts of nuclear deformation and of deformed mean field potential have been extremely successful, and have allowed to make enormous progress in our understanding of nuclei ever after the pioneering works of Rainwater<sup>[1]</sup>, Bohr and Mottelson<sup>[2]</sup>, and Nilsson<sup>[3]</sup>. A basic notion that comes along with those concepts is that, for open shell nuclei, single-nucleon states are not eigenstates of angular momentum, or in short, that nucleons move in deformed orbitals. In spite of the rich phenomenology, too vast to be quoted here, up to date understood on the basis of these concepts, deformed orbitals have not yet been “seen”. While spherical orbitals have clearly been seen in electron induced one nucleon knock-out reactions<sup>[4]</sup>, there is no such a direct evidence of the existence of deformed orbitals.

In a spherical nucleus, each single-particle momentum distribution  $n^{\{n\ell j\}}(p)$  is characteristic of the spherical orbital  $\{n\ell j\}$  occupied by the nucleon. Single-particle momentum distributions are probed in one nucleon knock-out reactions and, as already indicated, in the past years very precise coincidence  $(e, e'N)$  measurements have made possible to map out the momentum distributions of various spherical bound orbitals<sup>[4, 5]</sup>. An interesting question is whether such experiments can also measure momentum distributions of deformed orbitals.

In a deformed nucleus the momentum distribution of a given deformed orbital  $i$  is in general a linear combination of spherical orbitals ( $n^i(p) = \sum_{\ell j} c_{\ell j}^i n_{\ell j}(p)$ ). The linear combination depends on the deformation of the mean field and on the particular orbital  $i$ . Thus, for each orbital  $i$  and mean field deformation there is a characteristic momentum distribution that can in principle be identified by these reactions. Indeed, some of the existing experimental data on  $(e, e'p)$  from  $^{28}\text{Si}$  and  $^{146}\text{Nd}$  can be explained on the basis of the deformed model<sup>[6, 7]</sup>. In particular, a comparison of data on quasielastic electron scattering from  $^{142}\text{Nd}$  and from  $^{146}\text{Nd}$ <sup>[7, 8]</sup> confirmed the presence of deformation effects that had been discussed in Refs. [9, 10]. As pointed out in these references, for even-even targets the main effects of deformation in  $(e, e'p)$  are related to the higher level density of low-lying states in the residual nucleus and to the fragmentation of strength of spherical orbitals. With even-even targets it is not possible to measure the total momentum distribution of the deformed orbital in a single transition, since for transitions to a well resolved discrete state of the residual nucleus only a given component

$\ell j$  of the deformed state  $i$  enters. Therefore the interpretation of data is not free of ambiguities. In this work we show that the most direct evidence for the existence of deformed orbitals can be gained by using deformed odd-A targets that can be polarized.

Recently we have investigated spin-dependent momentum distributions in odd-A deformed nuclei and spectral functions for quasielastic electron scattering with polarized beam and target<sup>[11, 12]</sup>. We have found that there are new features of nuclear structure and spin degrees of freedom that can be revealed in these reactions, and deserve further study. This paper is devoted to study the new features connected with the role of  $\ell$ -wave mixing in deformed single-particle orbitals.

We show that measuring response functions and/or asymmetries in  $\vec{A}(\vec{e}, e'p)B$  reactions for specific transitions in deformed nuclei one can see the  $\ell$ -wave mixing in deformed orbitals. To this end we show results for response functions and asymmetries for  $^{37}\vec{K}(\vec{e}, e'p)^{36}\text{Ar}$  and for  $^{37}\vec{Ar}(\vec{e}, e'n)^{36}\text{Ar}$ , where the residual nucleus is left in its first excited state. The target and residual nuclei are described within the scheme of the rotational model of Bohr and Mottelson<sup>[13]</sup>, using as intrinsic wave functions the Slater determinants obtained from density dependent Hartree-Fock (DDHF) calculations. We show the results obtained from DDHF calculations, and stress the role of nuclear deformation and  $\ell$ -wave mixing by comparison to results obtained in the spherical limit.

Most of the results presented here are based on the plane wave impulse approximation (PWIA) that neglects final state interactions (FSI), allowing to identify nuclear structure effects in a more transparent way. We also present a discussion of results obtained in DWIA (distorted wave impulse approximation) to illustrate the fact that the effect of deformation discussed here is quite different from standard FSI effects, and cannot be masked by them.

This paper has the following structure. In section II we give a brief summary of theory and details of calculations. In section III we show and discuss the results on response functions and polarization observables for  $^{37}\vec{K}(\vec{e}, e'p)^{36}\text{Ar}$  and  $^{37}\vec{Ar}(\vec{e}, e'n)^{36}\text{Ar}$ , obtained in PWIA with the deformed (DDHF) wave functions. In section IV we show the corresponding results on response functions and asymmetries obtained in the spherical limit and compare PWIA and DWIA results. Our final remarks are summarized in section V.

## 2 Brief summary of theory and details of calculations

### 2.1 $\vec{A}(\vec{e}, e'p)\text{B}$ reactions

The general formalism of quasielastic scattering from complex nuclei with polarized beam and target has been presented in several works<sup>[12],[14]–[16]</sup>. Here we follow the conventions and notations of Donnelly and collaborators<sup>[12, 14, 15]</sup> and summarize briefly the equations of interest to describe processes of the type  $\vec{A}(\vec{e}, e'N)\text{B}$  for transitions to discrete states  $J_B^\pi$  in the residual nucleus. Here we mainly reformulate some of the equations given in past works to make the formalism more suitable to discuss the points of interest in this paper.

We use kinematical variables in the laboratory frame as follows:  $k^\mu = (\epsilon, \mathbf{k})$  and  $k'^\mu = (\epsilon', \mathbf{k}')$  are initial and final electron four-momenta;  $P_A^\mu = (M_A, \mathbf{0})$ ,  $P_B^\mu = (E_B, \mathbf{P}_B)$  and  $p_N^\mu = (E_N, \mathbf{p}_N)$  are the four-momenta of the target nucleus, residual nucleus, and outgoing nucleon, respectively;  $Q^\mu = (\omega, \mathbf{q}) = k^\mu - k'^\mu = p_N^\mu + P_B^\mu - P_A^\mu$  is the four-momentum transferred by the virtual photon. The laboratory reference frame  $(x, y, z)$  is defined by the unit vectors

$$\mathbf{e}_z = \frac{\mathbf{q}}{q} \quad \mathbf{e}_y = \frac{\mathbf{k}_i \times \mathbf{k}_f}{k_i k_f \sin \theta_e} \quad \mathbf{e}_x = \mathbf{e}_y \times \mathbf{e}_z \quad (1)$$

Following Refs. [12, 14] we write the differential cross section for the  $\vec{A}(\vec{e}, e'p)\text{B}$  process as

$$\frac{d\sigma}{d\epsilon' d\Omega_e d\Omega_N} = \Sigma + h\Delta = \Sigma_0 [1 + P_\Sigma + hP_\Delta] \quad (2)$$

where  $h$  is the helicity of the longitudinally polarized electron, and  $\Sigma_0$  is the differential cross section for unpolarized beam and target.  $P_\Sigma$  depends on the target polarization and  $P_\Delta$  contributes for polarized beam and target. Note that for unpolarized target  $P_\Delta$  is zero in PWIA, but in general it is not necessarily zero<sup>[16]–[18]</sup>. Using current conservation to separate transverse and longitudinal components of the electron and nuclear currents, the following standard expressions are obtained for the helicity sum ( $\Sigma$ ) and the helicity difference ( $\Delta$ ) cross sections

$$\Sigma = \mathcal{K} \sum_X V_X R^X \quad X = L, T, LT, TT \quad (3)$$

$$\Delta = \mathcal{K} \sum_{X'} V_{X'} R^{X'} \quad X' = LT', T' \quad (4)$$

where  $X$  runs over the four response functions longitudinal (L), transverse (T), transverse–longitudinal (LT) and transverse–transverse (TT) interferences, and  $X'$  runs over the other two possible interference response functions, transverse–longitudinal (LT') and transverse–transverse (T'). The kinematical factor  $\mathcal{K}$  is given by

$$\mathcal{K} = \sigma_M \frac{p_N M_N M_B}{(2\pi)^3 M_A f_{\text{rec}}}, \quad (5)$$

with  $\sigma_M$  and  $f_{\text{rec}}$  the standard Mott cross section and recoil factor, respectively. Explicit expressions for the other kinematical factors  $V_X$  and  $V_{X'}$  can be found in Ref. [14]. Here we only recall that these kinematical factors have different dependences on the electron kinematics, which together with the various dependences on polarization and outgoing nucleon directions, allow in principle to separate the different response functions.

In PWIA, each of the hadronic response functions  $R^X$ ,  $R^{X'}$  can in turn be factored into a part that depends on the nuclear structure and a part that depends on the half–off–shell nucleon current:

$$R^X = \mathcal{R}^X \overline{M}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*) \quad (6)$$

$$R^{X'} = \mathcal{R}^{X'} \cdot \widehat{\mathbf{M}}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*) \quad (7)$$

The single–nucleon responses  $\mathcal{R}^X$ ,  $\mathcal{R}^{X'}$  have been studied in Ref. [15] for two different choices (cc1 and cc2) of the half–off–shell nucleon current operator, as well as for different prescriptions concerning current conservation. For instance, once cc1 is chosen one may impose current conservation and eliminate the third component of the single–nucleon current in favour of the zero component. This prescription is called cc1<sup>(0)</sup>. On the contrary, one may eliminate the zero component in favour of the third (cc1<sup>(3)</sup>), or one may simply calculate independently the four components without imposing current conservation (ncc1). Of course for a free nucleon the same result is obtained with any of these prescriptions, but in quasielastic scattering the initial nucleon is not on shell, and different prescriptions may lead to different results. We shall come back to this point in section II.3.

The dependence on the nuclear structure in eqs. (6) and (7) is contained in the scalar and vector momentum distributions  $\overline{M}$  and  $\widehat{\mathbf{M}}$ , that depend

not only on the momentum  $\mathbf{p}$  ( $= -\mathbf{P}_B$ ) of the struck nucleon but also on the target polarization  $\mathbf{P}^*$ , and on the particular nuclear transition ( $J_A \rightarrow J_B$ ) under consideration. The scalar and vector momentum distributions are defined in terms of the spin dependent momentum distribution matrix  $M_{\sigma\sigma'}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*)$  as follows<sup>[11]</sup>

$$\overline{M}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*) = \text{Tr} [M^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*)] \quad (8)$$

$$\widehat{\mathbf{M}}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*) = \text{Tr} [\boldsymbol{\sigma} M^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*)] \quad (9)$$

where traces are taken in spin space, and the matrix  $M_{\sigma\sigma'}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*)$  is related to the spin dependent polarized spectral function defined in Ref. [12] by

$$S_{\sigma\sigma'}(\mathbf{p}, E; \mathbf{P}^*) = \sum_{J_B} \delta(E - M_A^0 + E_{J_B}) M_{\sigma\sigma'}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*) \quad (10)$$

or

$$M_{\sigma\sigma'}^{(J_A \rightarrow J_B)}(\mathbf{p}, \mathbf{P}^*) = \sum_{M_B} \langle J_A J_A(\Omega^*) | a_{\mathbf{p}\sigma}^+ | J_B M_B \rangle \langle J_B M_B | a_{\mathbf{p}\sigma'} | J_A J_A(\Omega^*) \rangle \quad (11)$$

Here and in what follows we assume for simplicity that the target is 100% polarized in the direction  $\mathbf{P}^*$ , characterized by the angles  $\Omega^*$ . Hence the target has angular momentum projection  $M_A = J_A$  in this direction.  $\sigma$  and  $\sigma'$  denote spin projections of the struck nucleon on the laboratory  $z$ -axis.

To match with the notation in Refs. [12, 14, 15] we denote by  $l$  (longitudinal) the vector components of  $\mathbf{R}^{X'}$  and  $\widehat{\mathbf{M}}^{(J_A \rightarrow J_B)}$  along  $\mathbf{q}$  ( $z$ -axis), and by  $s$  (side-ways), and  $n$  (normal) the vector components in the perpendicular ( $x, y$ ) plane.

From eqs. (2) to (9) it can be seen that the vector momentum distribution only contributes for polarized beam and target. Furthermore, in Ref. [11] it was shown that this new momentum distribution depends only on the odd-nucleon wave function and can be probed in transitions to the ground state rotational band of the residual nucleus. Therefore we focus on transitions to states  $J_B$  in the ground state band ( $K_B^\pi = 0^+$ ) of the residual nucleus.

General expressions for spectral functions and momentum distributions for arbitrary target polarization can be found in Refs. [11, 12]. Most of the results shown here are for 100% polarized target in the  $z$ -direction ( $\mathbf{P}^* \parallel \mathbf{q}$ ).

In this particular case, the scalar and vector momentum distributions are given by

$$\overline{M}^{(J_A \rightarrow J_B)} = \sum_{\lambda \text{ even}} P_\lambda(\cos \theta_p) \sum_{\ell j \ell' j'} \mathcal{F}^\lambda(\ell j; \ell' j') \tilde{\phi}_{\ell j}(p) \tilde{\phi}_{\ell' j'}(p) \quad (12)$$

$$\widehat{M}_s^{(J_A \rightarrow J_B)} = \cos \phi_p \widehat{M}_t^{(J_A \rightarrow J_B)} \quad (13)$$

$$\widehat{M}_n^{(J_A \rightarrow J_B)} = \sin \phi_p \widehat{M}_t^{(J_A \rightarrow J_B)} \quad (14)$$

$$\widehat{M}_{\begin{bmatrix} l \\ t \end{bmatrix}}^{(J_A \rightarrow J_B)} = \sum_{L \text{ even}} \left[ \begin{array}{c} P_L(\cos \theta_p) \\ P_L^1(\cos \theta_p) \end{array} \right] \sum_{\ell j \ell' j'} \mathcal{G}_{\begin{bmatrix} l \\ t \end{bmatrix}}^L(\ell j; \ell' j') \tilde{\phi}_{\ell j}(p) \tilde{\phi}_{\ell' j'}(p) \quad (15)$$

with  $p$ ,  $(\theta_p, \phi_p)$  the absolute value and direction of the momentum  $\mathbf{p}$  of the bound nucleon.  $\tilde{\phi}_{\ell j}(p)$  and  $\tilde{\phi}_{\ell' j'}(p)$  are the spherical wave projections of the deformed odd-nucleon wave function in momentum space (see section II.2), and the coefficients  $\mathcal{F}$ ,  $\mathcal{G}$  are given by:

$$\begin{aligned} \mathcal{F}^\lambda(\ell j; \ell' j') &= \frac{\hat{\lambda}^2 \hat{J}_A^2 \hat{J}_B^2 \hat{\ell} \hat{\ell}' \hat{j} \hat{j}'}{2\pi} (-1)^{J_A + j + j' - \frac{1}{2}} \begin{pmatrix} J_A & J_A & \lambda \\ -J_A & J_A & 0 \end{pmatrix} \begin{pmatrix} J_B & J_A & j \\ 0 & K_A & -K_A \end{pmatrix} \\ &\quad \begin{pmatrix} J_B & J_A & j' \\ 0 & K_A & -K_A \end{pmatrix} \begin{pmatrix} \ell & \ell' & \lambda \\ 0 & 0 & 0 \end{pmatrix} \left\{ \begin{array}{ccc} j' & j & \lambda \\ \ell & \ell' & \frac{1}{2} \end{array} \right\} \left\{ \begin{array}{ccc} j' & j & \lambda \\ J_A & J_A & J_B \end{array} \right\} \quad (16) \end{aligned}$$

$$\begin{aligned} \mathcal{G}_{\begin{bmatrix} l \\ t \end{bmatrix}}^L(\ell j; \ell' j') &= -\frac{\sqrt{3}}{\pi} \hat{L}^2 \hat{J}_A^2 \hat{J}_B^2 \hat{\ell} \hat{\ell}' \hat{j} \hat{j}' (-1)^{J_A + \ell' + j} \sum_{\lambda=L \pm 1} \hat{\lambda}^2 \begin{pmatrix} J_A & J_A & \lambda \\ -J_A & J_A & 0 \end{pmatrix} \\ &\quad \begin{pmatrix} J_B & J_A & j \\ 0 & K_A & -K_A \end{pmatrix} \begin{pmatrix} J_B & J_A & j' \\ 0 & K_A & -K_A \end{pmatrix} \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{pmatrix} \left\{ \begin{array}{ccc} L & \lambda & 1 \\ \ell' & j' & \frac{1}{2} \\ \ell & j & \frac{1}{2} \end{array} \right\} \\ &\quad \left\{ \begin{array}{ccc} j' & j & \lambda \\ J_A & J_A & J_B \end{array} \right\} \left[ \begin{array}{c} -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & L & \lambda \\ 0 & 0 & 0 \end{pmatrix} \\ \frac{1}{\sqrt{L(L+1)}} \begin{pmatrix} 1 & L & \lambda \\ -1 & 1 & 0 \end{pmatrix} \end{array} \right] \quad (17) \end{aligned}$$

We would like to remark that the sum over  $\ell j, \ell' j'$  is characteristic of nucleon knock-out from a deformed orbital. When the struck nucleon is in a spherical orbital  $\{n\ell j\}$  this sum reduces to a single term.

It is easy to check that for  $J_B = 0$  (*i.e.*, for transitions to the ground state of the residual nucleus) the coefficients  $\mathcal{F}^\lambda(\ell j; \ell' j')$  and  $\mathcal{G}_{l,t}^L(\ell j; \ell' j')$  are zero unless  $j = j' = J_A$ . This means that for transitions to the ground state in the residual nucleus only one component ( $\ell j = \ell' j'$ ) contributes. For the nuclei under consideration only the  $d_{\frac{3}{2}}$  component would contribute. Hence this transition ( $J_A^\pi = \frac{3}{2}^+ \rightarrow J_B^\pi = 0^+$ ) allows us to see only one  $\ell j$ -component of the deformed orbital, and it is more interesting to consider the transition to the first excited state ( $2^+$ ) that, as we shall see, allows us to explore the deformed orbital in its full complexity. In what follows we focus on the transition  $J_A^\pi = \frac{3}{2}^+ \rightarrow J_B^\pi = 2^+$ .

As it is discussed in the next section, for the two target nuclei considered here the ground state rotational band has  $J_A = \frac{3}{2}$  and  $K_A = \frac{1}{2}$ . The coefficients  $\mathcal{F}$  and  $\mathcal{G}$  for these  $J_A$ ,  $K_A$  values and for  $J_B = 2$  are given in tables I–III.

For unpolarized target only the  $\lambda = 0$  multipole of the scalar momentum distribution ( $\overline{M}_{\lambda=0}^{(J_A \rightarrow J_B)}$ ) contributes to the differential cross section  $\Sigma_0$ .

The notation for momentum distributions used here follows that of Ref. [11], and is closely related to that used in Ref. [12] where we used the notation  $N_0^{J_B}$ ,  $N_l^{J_B}$ ,  $N_s^{J_B}$  and  $N_n^{J_B}$  in place of  $\overline{M}^{(J_A \rightarrow J_B)}$ ,  $\widehat{M}_l^{(J_A \rightarrow J_B)}$ ,  $\widehat{M}_s^{(J_A \rightarrow J_B)}$  and  $\widehat{M}_n^{(J_A \rightarrow J_B)}$ , respectively. We would like to point out that the expressions given in appendix B of Ref. [12] are for arbitrary target polarization and polarization direction, as well as for arbitrary  $J_A \rightarrow J_B$  transition, while the ones given in eqs. (12) to (17) are for a 100% polarized target in the  $z$  direction and for a transition to the state  $J_B$  in the ground state rotational band of the residual nucleus.

## 2.2 Nuclear structure calculation

The scalar and vector momentum distributions defined in the previous section have been calculated with the wave functions obtained from DDHF calculations, assuming a time reversal invariant axially symmetric mean field. As in previous work on momentum distributions in deformed nuclei<sup>[9, 11, 12]</sup> we have used the McMaster<sup>[19]</sup> version of the HF program, and the SKA interaction<sup>[20]</sup>. The targets and residual nuclei are described by the same mean field. We have ignored pairing correlations that cannot be reliably handled by BCS approximation in these *sd* shell nuclei. The calculated self-



consistent mean field is oblate ( $Q_0 < 0$ ) and the odd nucleon (a neutron in  $^{37}\text{Ar}$  and a proton in  $^{37}\text{K}$ ) is in a  $K^\pi = \frac{1}{2}^+$  state, and has basically the same wave function in both cases. This can be seen in table IV, where we give the weights of the  $\ell j$ -projection of the odd-nucleon wave function in both cases.

In table V we show the theoretical results obtained for the ground state binding energies, charge r.m.s radii, and magnetic moments, as well as for intrinsic quadrupole moments, moments of inertia and decoupling parameters. The properties of the rotational bands are calculated according to the expressions given in Ref. [21], in particular, we use the cranking formula for the moments of inertia. The available experimental data are also given in the table. It is also worth pointing out that data on r.m.s. radii<sup>[22]</sup> and quadrupole moments<sup>[23, 24]</sup> for other neighbour nuclei like  $^{37}\text{Cl}$  and  $^{39}\text{K}$  are close to the theoretical results in table V. In particular, the negative values of the intrinsic quadrupole moments are in good agreement with the experimental<sup>[23, 24]</sup> quadrupole moment for  $^{36}\text{Ar}$  ( $Q_0^{\text{exp}} = -38.5 \pm 21.0$ ).

As seen in table V we get good agreement with experiment for binding energies and magnetic moments. Also, the experimental spectral sequence<sup>[23]</sup>  $\left(\frac{3}{2}^+, \frac{1}{2}^+, \frac{7}{2}^+, \frac{5}{2}^+\right)$  of angular momentum states in  $^{37}\text{Ar}$  and  $^{37}\text{K}$  is well reproduced with the moments of inertia and decoupling parameters in table V, although the first excited state ( $\frac{1}{2}^+$ ) is lower than the experimental  $\frac{1}{2}^+$  state. Actually, the  $\frac{7}{2}^+$  states that we get at  $E \sim 2.6$  MeV in both  $^{37}\text{Ar}$  and  $^{37}\text{K}$ , correspond quite nicely with the experimental  $\frac{7}{2}^+$  state at 2.22 MeV in  $^{37}\text{Ar}$  and with a state at 2.28 MeV in  $^{37}\text{K}$  whose spin and parity have not yet been assigned. Also the  $\frac{5}{2}^+$  states at 2.80 and 2.75 MeV in  $^{37}\text{Ar}$  and  $^{37}\text{K}$ , respectively, are fairly well reproduced theoretically (with the  $\mathcal{I}$  and  $a$  values in table V we get  $E \sim 2.85$  and 2.86 MeV, respectively). The  $2_1^+$  state in  $^{36}\text{Ar}$  is also close to the one we get with the moment of inertia  $\mathcal{I}^{\text{th.}} = 2.5 \text{ MeV}^{-1}$ . In summary, our DDHF calculations give a fairly accurate description of the properties of the ground state bands of the nuclei involved in the transitions under study.

Coming back to the calculations of momentum distributions, as seen from eqs. (12) to (15), for the transitions considered all we need is to compute the spherical  $\ell j$ -wave projection of the deformed odd-nucleon wave function in momentum space. The latter are obtained as follows. From the Hartree-Fock output we get the odd-nucleon wave function in coordinate space and cylindrical coordinates with intrinsic angular momentum projection  $K = \frac{1}{2}$

and parity  $\pi = +1$ :

$$\Phi(\mathbf{r}, s, q) = \chi_\tau(q) \sum_{\alpha} C_{\alpha} \varphi_{\alpha}(\mathbf{r}, s) \quad (18)$$

where  $\chi_\tau(q)$  denotes isospin function and where  $\alpha$  denotes the set of quantum numbers  $\alpha = [n_r, n_z, \Lambda, \Sigma]$  with  $\Lambda + \Sigma = K = \frac{1}{2}$ ,  $\Lambda \geq 0$  and  $2n_r + n_z + \Lambda = N = \text{even}$ . The sum extends over all basis states from  $N = 0$  to  $N = 10$ .  $C_{\alpha}$  are the HF coefficients and  $\varphi_{\alpha}$  are the basis wave functions<sup>[25]</sup> normalized to 1.

Taking the Fourier transform in eq. (18), we get the odd-nucleon wave function in momentum space

$$\tilde{\Phi}(\mathbf{p}, s, q) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d\mathbf{r} e^{-i\mathbf{p}\cdot\mathbf{r}} \Phi(\mathbf{r}, s, q) \quad (19)$$

from which the  $\ell j$ -projection is obtained as

$$\tilde{\phi}_{\ell j}(p) = \langle \mathcal{Y}_{\ell j}^{m_j=\frac{1}{2}}(\hat{\mathbf{p}}, s) | \tilde{\Phi}(\mathbf{p}, s) \rangle \quad (20)$$

where the brackets indicate integration over  $\mathbf{p}$ -direction and spin, and where isospin labels and functions have been omitted to simplify the notation. Using the standard definition of the vector spherical harmonics

$$\mathcal{Y}_{\ell j}^{m_j}(\hat{\mathbf{p}}, s) = \sum_{m\sigma} \langle \ell m \frac{1}{2} \sigma | j m_j \rangle Y_{\ell}^m(\hat{\mathbf{p}}) \chi_{\sigma}(s) \quad (21)$$

we get

$$\tilde{\phi}_{\ell j}(p) = \sum_{\alpha} C_{\alpha} \langle \ell \Lambda \frac{1}{2} \Sigma | j \frac{1}{2} \rangle R_{\ell}^{\alpha}(p) \quad (22)$$

with

$$\begin{aligned} R_{\ell}^{\alpha}(p) &= (-i)^{N+2\Lambda} \sqrt{\pi_i} \left[ \frac{(2\ell+1)(\ell-\Lambda)!}{2(\ell+\Lambda)!} \right]^{\frac{1}{2}} \\ &\int_0^{\pi} \sin \theta_p d\theta_p P_{\ell}^{\Lambda}(\cos \theta_p) \psi_{n_{\perp}}^{\Lambda}(p \sin \theta_p) \psi_{n_z}(p \cos \theta_p) \end{aligned} \quad (23)$$

and

$$\psi_{n_{\perp}}^{\Lambda}(p_{\perp}) = \left[ \frac{2n_{\perp}!}{\beta_{\perp}^2 (n_{\perp} + \Lambda)!} \right]^{\frac{1}{2}} \eta^{\Lambda/2} e^{-\eta/2} L_{n_{\perp}}^{\Lambda}(\eta), \quad (24)$$

$$\psi_{n_z}(p_z) = \left[ \frac{1}{\beta_z \sqrt{\pi} 2^{n_z} n_z!} \right]^{\frac{1}{2}} e^{-\xi^2/2} H_{n_z}(\xi) \quad (25)$$

where  $L_{n_\perp}^\Lambda$  and  $H_{n_z}$  are associated Laguerre and Hermite polynomials respectively,  $\eta = p_\perp^2/\beta_\perp^2$ ,  $\xi = p_z/\beta_z$ , and  $\beta_\perp$ ,  $\beta_z$  are the inverse of the harmonic oscillator length parameters ( $\beta_{\perp,z} = \sqrt{m\omega_{\perp,z}}$ ). (Natural units  $c = \hbar = 1$  have been used throughout).

As seen in table IV for both  $^{37}\text{K}$  and  $^{37}\text{Ar}$  the dominant  $\ell j$ -wave component is the  $d_{\frac{3}{2}}$  ( $\sim 75\%$ ), but the  $s_{\frac{1}{2}}$  component is also important ( $\sim 19\%$ ). Of decreasing importance are the higher  $\ell j$  components  $d_{\frac{5}{2}}$  ( $\sim 5\%$ ) and  $g_{\frac{7}{2}}$  ( $\sim 0.7\%$ ), while even higher components contribute less than  $0.1\%$ .

### 2.3 Choice of kinematics and single-nucleon responses

We are interested in coincidence measurements in the quasifree region and in transitions to the lower states in the residual nucleus to probe essentially the odd-nucleon wave function. Our interest is to choose the kinematics in such a way that the kinetic energy of the outgoing nucleon is large enough to minimize effects of final state interactions, and yet the cross sections are as large as possible. Though for comparison with data, FSI have to be taken into account (see for instance Ref. [4]), at sufficiently high momentum transfer, and for an energy transfer at the quasielastic peak, the process is expected to be less influenced by final state interactions and exchange effects. Hence we select the kinematics by fixing the value of the momentum transfer  $q$  to be reasonably large below 1 GeV. For the selected  $q$  value, we choose the energy transfer  $\omega$  to be that at the quasielastic peak

$$\omega = \omega_{\text{q.p.}} = \sqrt{q^2 + M^2} - M + E_s \quad (26)$$

where  $E_s$  is the neutron (proton) separation energy in  $^{37}\text{Ar}$  ( $^{37}\text{K}$ ). Solving the energy balance equation

$$\sqrt{p_N^2 + M^2} + \sqrt{p^2 + M_B^2} = M_A + \omega \quad (27)$$

for  $p_N = |\mathbf{q} - \mathbf{p}|$ , and subject to the condition  $-1 \leq \cos \theta_p \leq 1$ , one can obtain the range of allowed values of the bound nucleon momentum  $p_{\min} \leq p \leq p_{\max}$ .

For fixed values of  $q$  and  $\omega$  as well as fixed mass  $M_B$  of the residual nucleus one can vary the bound nucleon momentum  $p$  in the interval between  $p_{\min}$  and  $p_{\max}$  by varying the angle  $\theta_N$ . For the cases of interest here in which the mass of the residual nucleus is close to the ground state mass ( $M_B = 33.5$  GeV  $\gg q$ ) one has that the allowed range of variation for  $p$  is between  $p_{\min} = 0$  and  $p_{\max} \sim 2q$ .

Of course as  $\theta_N$  varies not only  $p$  varies, but also  $p_N$  has to change to obey energy–momentum conservation. However, from the expression for  $p_N$

$$p_N = p_N(p) = \left[ \left( \omega + M_A - \sqrt{p^2 + M_B^2} \right)^2 - M^2 \right]^{\frac{1}{2}} \quad (28)$$

one sees that the variation of  $p_N$  is negligible, since one has that  $M_B^2 \gg p^2$  in the whole range of variation of  $p$ . The maximum value of  $p_N$  ( $p_N = q$  for  $p = 0$ ) decreases at most 1.5% in going from  $p = p_{\min}$  to  $p = p_{\max}$  for  $q = 500$  MeV. Hence FSI effects that depend on  $p_N$  remain essentially constant for this kinematics. It should be remarked that in this kinematics  $\theta_p$  and  $p$  are no longer independent but are related by

$$\cos \theta_p = \cos \theta_p(p) = \frac{p_N^2(p) - q^2 - p^2}{2pq} \quad (29)$$

The single–nucleon responses  $\mathcal{R}^X$ ,  $\mathcal{R}^{X'}$  for this kinematics and assuming  $\phi_N = 0$  are represented in Figs. 1 and 2, respectively, for proton (a) and neutron (b) emission and for  $q = 500$  MeV. The figures represent the results for the choice cc1<sup>(0)</sup>. At the kinematics considered, results for cc2<sup>(0)</sup>, ncc1 and ncc2 are similar to the ones in Figs. 1 and 2. Only the choices cc1<sup>(3)</sup> and cc2<sup>(3)</sup>, which are less reliable, give different single–nucleon cross sections<sup>[15]</sup> in the  $p$ –range considered here.

We have chosen  $\phi_N = 0$  and hence the normal components of the  $\mathcal{R}^{X'}$  vectors are zero ( $\mathcal{R}_n^{LT'} = \mathcal{R}_n^{T'} = 0$ ). For  $\mathcal{R}^{LT'}$  the longitudinal components ( $\mathcal{R}_l^{LT'}$ ) start at zero and increase rapidly with increasing  $p$  values, while the side–ways components ( $\mathcal{R}_s^{LT'}$ ) remain practically constant (taking much larger absolute values in the odd proton case). For  $\mathcal{R}^{T'}$  the opposite is true, the  $l$  components remain practically constant while the  $s$  component for the proton increases rapidly starting from zero (in the neutron case it remains close to zero in the whole  $p$ –range). The scalar response functions  $\mathcal{R}^X$  are all practically constant with  $p$ , except  $\mathcal{R}^{LT}$  that starts at zero and

increases rapidly in absolute value with  $p$ . Of course in the neutron case only  $\mathcal{R}^T$  and  $\mathcal{R}^{LT}$  take large values, while in the proton case the large response functions are  $\mathcal{R}^L$ ,  $\mathcal{R}^T$  and  $\mathcal{R}^{LT}$ . The response  $\mathcal{R}^{TT}$  is in all cases very small. These considerations have to be kept in mind to understand to what extent the observable response functions ( $R^X$  and  $R^{X'}$ ) can reveal nuclear structure properties. In particular, the fact that  $\mathcal{R}^L$  and  $\mathcal{R}^T$  present a small variation with  $p$  justifies the approximation made in the context of  $y$ -scaling for inclusive unpolarized electron scattering<sup>[26]</sup>. The strong  $p$ -dependence of  $\mathcal{R}^{LT}$  indicates that the phenomenon of  $y$ -scaling may not hold in more general situations.

The similarity between  $\mathcal{R}^T$  and  $\mathcal{R}^L$  in the proton case is characteristic of our kinematics. For larger  $q$  values, for instance for  $q = 800$  MeV,  $\mathcal{R}^T$  is larger than  $\mathcal{R}^L$  in both proton and neutron cases. Otherwise, at  $q = 800$  MeV the single-nucleon response functions show the same qualitative features seen in Figs. 1 and 2, but quantitatively they are typically 2–3 times smaller, hence giving rise to smaller cross sections. In next sections all the results shown are for  $q = 500$  MeV and for  $cc1^{(0)}$ .

### 3 Response functions and polarization observables

In this section we present our results on response functions and polarization observables and discuss the role of the  $\ell$ -wave mixing in the deformed nucleon wave function. To this end we present first in Fig. 3 the scalar ( $\overline{M}$ ) and vector components ( $\widehat{M}_l$ ,  $\widehat{M}_t$ ) of the spin dependent momentum distributions defined in eqs. (12)–(15) for the transition from the ground state in  $^{37}\text{K}$  to the  $2^+$  state in the ground state band of  $^{36}\text{Ar}$ . The results for the transition from the ground state in  $^{37}\text{Ar}$  are similar. The coefficients  $\mathcal{F}$  and  $\mathcal{G}$  for this transition are tabulated in tables I–III.

$\overline{M}$ ,  $\widehat{M}_l$  and  $\widehat{M}_t$  are shown in plots (a), (b) and (d) of Fig. 3, respectively. Note that these momentum distributions have in principle two independent variables  $p$  and  $\theta_p$ , however the plots in Fig. 3 correspond to the kinematics discussed in section II.3 where  $\theta_p$  is a function of  $p$  (see eq. (29)). In plot (c) of Fig. 3 we show for comparison the  $\lambda = 0$  multipole of the scalar momentum distribution  $\overline{M}_{\lambda=0}$ , which is the only one contributing when the target is un-

polarized. As seen in tables I–III only the diagonal terms  $\ell j = \ell' j'$  contribute to  $\overline{M}_{\lambda=0}$ , while  $\overline{M}_{\lambda=2}$  and the vector momentum distribution components  $\widehat{M}_l$ ,  $\widehat{M}_t$  contain also important off-diagonal contributions. Depending on the  $p$  value and on the particular components of the spin dependent momentum distribution one has dominance of a given  $\ell j$  component or dominance of the interference terms with  $\ell j \neq \ell' j'$ . It should be stressed that the interference terms are only present for a polarized deformed target. In Fig. 3 we also show, for each momentum distribution component, the contributions from the most important terms. Looking at tables I–IV it is easy to find out that in general the main contributions are from the terms with  $\ell = \ell' = 0$  ( $ss$ ),  $\ell = \ell' = 2$  ( $dd$ ), and  $\ell = 0, \ell' = 2$  or viceversa ( $sd$ ).

As seen in Fig. 3(a) the scalar momentum distribution has a maximum at  $p = 0$  due to the  $ss$  contribution that dominates in the low  $p$  region ( $p \leq 0.5 \text{ fm}^{-1}$ ), and a second maximum at  $p \approx .8 \text{ fm}^{-1}$  that is made up not only from the  $dd$  contribution but also from the contributions of  $sd$  and  $ss$  terms. Though the scalar momentum distribution in the polarized case has a more pronounced second maximum than in the unpolarized case (compare plots (a) and (c) in Fig. 3), the profiles are not very different in the two cases. On the contrary the vector momentum distribution components  $l$ ,  $t$  have quite different profiles. Particularly,  $\widehat{M}_t$  is dominated by the  $sd$  interference term, and therefore contains basically the information missing in the unpolarized momentum distribution. The shape of  $\widehat{M}_t$  is characteristic of the  $sd$  interference term and its observation would provide the most direct evidence for the  $\ell$ -wave mixing in deformed orbitals.

In Figs. 4 and 5 we show the response functions for the reactions  $^{37}\vec{\text{K}}(\vec{e}, e'p)^{36}\text{Ar}$  and for  $^{37}\vec{\text{Ar}}(\vec{e}, e'n)^{36}\text{Ar}$ , respectively. Also shown for comparison by dashed lines, are the response functions for unpolarized targets. As seen in the figures the profiles of the response functions reflect quite nicely the features of the momentum distributions shown in Fig. 3, and depend strongly on the  $\ell$ -mixing. Clearly, the relationship between the results in Fig. 4 and those in Fig. 5 is dictated by the differences in the single-proton and neutron response functions depicted in Figs. 1 and 2.

The response functions  $R^X$  depend on the scalar momentum distribution and follow closely the profiles in Figs. 3(a) and 3(c) for the polarized and unpolarized cases, respectively. This is so particularly for the longitudinal ( $R^L$ ) and transverse ( $R^T$ ) response functions for which the corresponding

single-nucleon responses are fairly constant with  $p$  (see Fig. 1). As already remarked, at the kinematics considered here, the single-proton responses  $\mathcal{R}^L$  and  $\mathcal{R}^T$  are very close and therefore the total  $R^L$ ,  $R^T$  responses in Fig. 4 are practically identical, while in Fig. 5  $R^T$  is much larger than  $R^L$ . The single-nucleon response  $\mathcal{R}^{LT}$  starts at zero and then goes fast to large negative values. Hence, the response function  $R^{LT}$  produces a quite different mapping than  $R^L$  and  $R^T$  of the scalar momentum distribution. The same can be said about  $R^{TT}$  which is much smaller than  $R^{LT}$ , and, in this sense, is less interesting.

As seen from the comparison to the unpolarized case, the main information that these responses  $R^X$  carry is that corresponding to the  $ss$  and  $dd$  contributions and not much more information seems to be gained polarizing the target. It is clear that in these response functions the main observational feature due to deformation is the presence of the two sizeable peaks, one corresponding to the  $s$ -wave and the other corresponding to the  $d$ -wave. This feature, which is characteristic of the deformed orbital, has already been observed to some extent in quasielastic scattering from deformed even-even targets[6, 7]. The important difference is that for even-even targets the observation of the effect is associated to lack of resolution, whereas in the case of odd-A target this feature can be observed in a transition to a single state in the residual nucleus, as considered here. Of course, when the struck nucleon is in a single spherical orbital there is no way to observe such a pronounced double-peaked structure in a single transition. This point is illustrated in Section IV where we discuss the spherical limit.

Even more unambiguous evidence on the  $\ell$ -mixing in the deformed orbitals can be gained by measuring the response functions for polarized beam and target,  $R^{LT'}$  and  $R^{T'}$ . As seen in Figs. 4 and 5 these response functions are more sensitive to the  $sd$  interference terms, particularly  $R^{LT'}$  whose shape is basically that of the  $sd$  contribution to the transverse vector momentum distribution (see Fig. 3(d)).  $R^{LT'}$  is dominated by the  $sd$  contribution while  $R^{T'}$  is not. This follows from the facts that *i)*  $\widehat{M}_t$  is dominated by the  $sd$  contribution but  $\widehat{M}_l$  is not, and *ii)* in the  $LT'$  response the transverse component of the single-nucleon response ( $\mathcal{R}_s^{LT'}$ ) dominates over the longitudinal one (for the odd-neutron case this is only true at low  $p$ ), while in the  $T'$  response the longitudinal single-nucleon response ( $\mathcal{R}_l^{T'}$ ) dominates. Thus the new feature of  $sd$  interference, a sign of the deformation of the orbital of the struck nucleon, can in principle be detected by measuring the  $R^{LT'}$  response

at the corresponding kinematics.

It should be emphasized that the choices of kinematics and of polarization direction play an important role in dictating which of the response functions ( $R^{T'}$  or  $R^{LT'}$ , or both) is more sensitive to the  $\ell$ -wave interference terms. We have carried out a search for the most favorable conditions to detect such  $sd$ -interference terms for the transitions of our concern here. We found that a particularly favorable case is that in which the target is polarized perpendicular to the scattering plane ( $\mathbf{P}^* \parallel \mathbf{e}_y$ ) and the outgoing nucleon is detected in the plane with  $\phi_N \sim 60^\circ$  (remaining kinematics as discussed in section II.3). In Fig. 6 we show the  $LT'$  and  $T'$  response functions for this particular case together with the  $ss$ ,  $sd$  and  $dd$  contributions. As seen in the figure, in this specific case the two  $R^{X'}$  response functions are large and are dominated by the  $sd$  contribution, hence providing a particularly interesting case for an experimental test.

We should emphasize however that also for  $\mathbf{P}^* \parallel \mathbf{e}_z$  and  $\phi_N = 0$  (the conditions to which Figs. 4 and 5 apply) the asymmetries  $\mathcal{P}_\Sigma$  and  $\mathcal{P}_\Delta$  take also quite sizeable values. As seen in Fig. 7,  $\mathcal{P}_\Sigma$  and  $\mathcal{P}_\Delta$  oscillate, depending on the  $p$ -value, between  $-0.4$  and  $+0.2$  and between  $-0.1$  and  $+0.5$ , respectively. These oscillations depend strongly on the nuclear structure. Actually  $P_\Sigma$  is independent of the single-nucleon responses and depends only on the scalar momentum distribution

$$\mathcal{P}_\Sigma = \frac{\Sigma - \Sigma_0}{\Sigma_0} = \frac{\overline{M}}{\overline{M}_{\lambda=0}} - 1. \quad (30)$$

As will be seen in the next section in the spherical limit  $P_\Sigma$  is just proportional to  $P_2(\cos \theta_p)$ . Hence the strong  $p$  dependence observed in the solid lines of Fig. 7 reflects the strong dependence of  $\mathcal{P}_\Sigma$  and  $\mathcal{P}_\Delta$  on the complex structure of the deformed orbital.

## 4 Comparison with spherical limit and considerations on FSI effects

To appreciate better the size of the effects of deformation, we compare here the results obtained in the previous section with the ones obtained in the spherical limit. It is also important to see whether FSI, that have not been



taken into account up to here, may produce effects in the response functions and asymmetries comparable to those produced by deformation.

For these purposes we present in this section PWIA and DWIA results obtained in the spherical limit, assuming that the struck nucleon is in a  $d_{\frac{3}{2}}$  orbital. The DWIA calculations have been made using a relativistic optical potential as described in Ref. [27]. A discussion of the effects of FSI in coincidence  $(\vec{e}, e'p)$  reactions from complex nuclei can be found in Refs. [16, 17], based on nonrelativistic and relativistic optical potentials, respectively. To our knowledge, FSI effects have not been studied yet for the case of  $(\vec{e}, e'n)$  reactions involving polarized complex nuclei. This study is complicated by the fact that optical potentials are not as well determined as they are in the proton case. Furthermore, one may have to take into account processes of the type  $(e, e'p)$  followed by  $(p, n)$ , especially if the outgoing neutron is not very energetic (*i.e.*, when the quasifree approximation may not be valid and charge-exchange processes may play a role). Since the neutron has a very small electric form factor, such issues can be relevant when the charge/longitudinal projections of the current are being examined. An estimate of the contribution of these charge-exchange processes to the unpolarized  $(e, e'n)$  cross section was calculated by Giusti and Pacati in Ref. [28]. In our case of an odd-neutron target this effect is not expected to be especially important as long as we restrict ourselves to missing energies corresponding to the neutron separation energy, as we do here. In this case, FSI effects are expected to be comparable to those for proton knockout, and therefore in this section we show only results for  $^{37}\vec{\text{K}}(\vec{e}, e'p)^{36}\text{Ar}$ .

The results shown in Fig. 8 correspond to the same transition and kinematics as those shown in Figs. 4–6; except that now we show only the results for the more representative response functions when the struck nucleon is a proton ( $^{37}\text{K}$  target) in a  $d_{\frac{3}{2}}$  orbital. For  $\mathbf{P}^* \parallel \mathbf{e}_z$  and  $\phi_N = 0$  we only show, in addition to the  $R^{X'}$  responses, the large  $R^X$  responses  $R^L$  and  $R^{LT}$ , because  $R^T$  is very similar to  $R^L$  for this kinematics and  $R^{TT}$  is much smaller (see also Fig. 4). The  $R^{X'}$  responses are also shown for  $\mathbf{P}^* \parallel \mathbf{e}_y$  and  $\phi_N = 60^\circ$  to compare with the results given in Fig. 6 for the deformed HF solution.

The effect of deformation stands out clearly when we compare the PWIA results in Fig. 8 (solid lines) with the corresponding results in Figs. 4 and 6. The complex and varied structures of the responses in Figs. 4 to 6 disappear in the spherical limit. In this limit the shape of all response functions is

similar, and has a single sizeable peak corresponding to that of the  $d_{\frac{3}{2}}$  wave function. This dominant feature, as seen in Fig. 8, prevails in the DWIA results (dashed lines). The same conclusion is reached if one assumes that the struck nucleon was in a single  $s_{\frac{1}{2}}$  orbital (in that case one has in addition that  $\widehat{M}_t = 0$  and the  $R^{X'}$  responses are tiny in most cases).

The effect of final state interactions, taken here into account in the distortion of the outgoing proton wave function produced by the nuclear optical potential, varies depending on the kinematics and on the various response functions, which in some cases can be strongly suppressed<sup>[2]</sup>. This agrees with the analyses carried out in Ref. [16], where one can also see that FSI effects on specific response functions can be minimized with appropriate choices of kinematics, decreasing with increasing energy of the outgoing nucleon. The FSI effects found here tend to be somewhat larger than those found in Ref. [16] because of the relativistic optical potential used here. A comparison between calculations with relativistic and nonrelativistic optical potentials can be found in Refs. [27, 29]. We do not enter in details here because the main purpose of the DWIA results shown in Fig. 8 is to illustrate that typical FSI effects are quite different from the effects of deformation. As discussed previously, using the deformed HF solution we found quite different shapes for the various  $R^X$  and  $R^{X'}$  responses that we could directly link to the specific role of  $\ell$ -wave mixing in each response function.

This conclusion also emerges from the comparison in Fig. 7 between the results for  $\mathcal{P}_\Sigma$  and  $\mathcal{P}_\Delta$  obtained with the deformed HF solution (solid lines), and the results obtained in the spherical limit in PWIA (dashed line) and in DWIA (short-dashed line). As seen in Fig. 7 the DWIA results obtained for  $\mathcal{P}_\Sigma$  and  $\mathcal{P}_\Delta$  are very close to the PWIA results over a fairly extended  $p$  range. In this range the effects of FSI are qualitatively (and in most cases quantitatively) similar in the polarized and in the unpolarized responses, and therefore, tend to cancel when calculating the asymmetries  $\mathcal{P}_\Sigma$  and  $\mathcal{P}_\Delta$ . On the contrary, the effects of deformation are different in the polarized and in the unpolarized responses, resulting in strong oscillations with  $p$  of  $\mathcal{P}_\Sigma$  and  $\mathcal{P}_\Delta$  that are in contrast with the smooth behaviours observed for the spherical limits. Note that in PWIA  $\mathcal{P}_\Sigma$  is independent of the single-nucleon responses, and that in the spherical limit eq. (30) reduces to a second order polynomial in  $\cos^2 \theta_p$

$$P_\Sigma = \frac{3}{5} P_2(\cos \theta_p), \quad (31)$$

with  $\theta_p$  given by eq. (29), which explains the smooth  $p$ -dependence of the dashed lines in Fig. 7. When FSI effects are included, eqs. (30) and (31) do not hold, but differences between DWIA and PWIA results in the spherical model are notably smaller than the differences between the PWIA results with deformed and spherical models.  $\mathcal{P}_\Delta$  depends on the single-nucleon responses even in PWIA, but in the  $p$  region of interest the dominant responses in the numerator and denominator have a very similar  $p$  dependence when one takes the spherical limit.

Obviously for comparisons to experimental data it would be desirable to dispose of results that take into account simultaneously both FSI and deformation effects. DWIA calculations using deformed HF solutions for the bound nucleon wave functions, are now in progress. The main difficulty in this task is that, for consistency, the existing DWIA codes have to be extended to take into account deformation in the optical potentials.

Nevertheless the present results indicate that the effects of deformation encountered here will prevail and be clearly identifiable when more realistic calculations beyond PWIA are made.

Thus, although DWIA calculations using the deformed model should be performed for a quantitative comparison to experimental data, the present results allow us to conclude that the effects of deformation will not be masked by those of FSI.

## 5 Summary and final remarks

The purpose of this work was to study the sensitivity to the  $\ell$ -wave mixing in deformed bound orbitals of response functions and asymmetries in  $\vec{A}(\vec{e}, e'N)B$  processes. For this purpose we choose to study the transitions from polarized  $^{37}\text{Ar}$  and  $^{37}\text{K}$  to the first  $2^+$  state in  $^{36}\text{Ar}$ , which we consider to be particularly interesting. The nuclear structure content of these two transitions is practically identical, while the different electromagnetic nature of the emitted particle (proton or neutron) in these transitions can in principle be used to probe and/or to emphasize different physics aspects. Of course, for experimental tests,  $^{37}\text{Ar}$  may be preferred over  $^{37}\text{K}$ , since the latter is very short-lived while the former is a noble gas.

The nuclear structure calculations were performed using the modern deformed mean field description provided by density dependent Hartree-Fock

(DDHF). The dependence on the nuclear structure of response functions and asymmetries comes through the scalar and vector momentum distributions for these transitions. We find that our DDHF results describe nicely the main properties of the (oblate,  $K^\pi = \frac{1}{2}^+$ ) ground state bands of these nuclei, and thus provide a reliable starting point for the calculation of the scalar and vector momentum distributions for the transitions considered. The kinematics and transitions are chosen to probe basically the odd-nucleon wave function of the target nuclei. Our DDHF results provide a deformed odd-nucleon orbital with important  $s$ - and  $d$ -wave components. This  $\ell$ -wave mixing is manifested in different characteristic ways in the scalar and vector momentum distribution, as well as in the polarized and unpolarized responses.

For unpolarized target only the monopole component of the scalar momentum distribution contributes. For polarized target the response functions are sensitive to the higher multipoles of the scalar momentum distribution and to the vector momentum distribution. We show that the latter (which can only be probed when the beam is polarized) depends strongly on the terms of interference between different  $\ell$ -wave components of the deformed bound wave function of the struck nucleon, predominantly interference between  $s$ - and  $d$ -waves for the cases studied. We also show how the characteristic  $sd$  interference shape is manifested in  $R^{X'}$  responses for different nucleon-detection plane and polarization direction. On the contrary, the scalar momentum distributions, and hence the  $R^X$  response functions, depend mainly on the incoherent sum of  $s$ - and  $d$ -wave contributions, showing a characteristic double-peaked structure. The asymmetries  $P_\Sigma$  and  $P_\Delta$  exhibit also a strong  $p$ -dependence due to the complex structure of the deformed orbital. We show that the characteristic shapes of response functions and asymmetries obtained for the deformed HF orbital disappear in the spherical limit. A comparison of PWIA and DWIA results in the spherical limit is presented to show that standard FSI and deformation effects are different in nature and manifest in a different manner.

Although for quantitative comparison with experimental data, DWIA calculations with the deformed model should be also performed, the results presented in sections III and IV allow us to conclude that FSI effects will not mask the effects of deformation discussed here. Hence, measuring response functions and/or asymmetries for these transitions with polarized deformed target and beam, would provide a unique tool to see the  $\ell$ -wave mixing in deformed bound orbitals.

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## Figure Captions

- **Fig. 1:** Single-nucleon responses  $\mathcal{R}^L$ ,  $\mathcal{R}^T$ ,  $\mathcal{R}^{LT}$  and  $\mathcal{R}^{TT}$  for proton (a) and neutron (b) emission. The choices of kinematics and single-nucleon current are as discussed in the text.
- **Fig. 2:** Same as Fig. 1 for the single-nucleon responses  $\mathcal{R}^{LT'}$  and  $\mathcal{R}^{T'}$ . Not shown are the normal components  $n$ , that are zero for the kinematics chosen ( $\phi_N = 0$ ).
- **Fig. 3:** Scalar (a), and vector longitudinal (b) and transverse (d) momentum distributions for the transition to the first  $2^+$  state in the residual nucleus  $^{36}\text{Ar}$  with polarized  $^{37}\text{K}$  target. Also represented is the momentum distribution for the same transition with unpolarized target (c). The dominant  $\ell\ell'$  contributions are shown by short-dashed ( $\ell = \ell' = 0$ ), dashed ( $\ell = 0, \ell' = 2$ ), and long-dashed ( $\ell = \ell' = 2$ ) lines.
- **Fig. 4:** Response functions for the reaction  $^{37}\text{K}(\vec{e}, e'p)^{36}\text{Ar}(2_1^+)$  with polarized target (solid lines) and with unpolarized target (dashed lines). Polarization direction parallel to  $\mathbf{q}$  and coplanar kinematics ( $\phi_N = 0$ ,  $\mathbf{P}^* \parallel \mathbf{e}_z$ ) have been chosen (see text).
- **Fig. 5:** Same as Fig. 4 for the reaction  $^{37}\text{Ar}(\vec{e}, e'n)^{36}\text{Ar}(2_1^+)$ .
- **Fig. 6:** Response functions  $R^{LT'}$  and  $R^{T'}$  for the same reaction as Fig. 4, but with polarization perpendicular to the scattering plane and out-of-plane kinematics ( $\phi_N = 60$ ,  $\mathbf{P}^* \parallel \mathbf{e}_y$ ). Also shown are the contributions from  $s$ -wave (short-dashed line),  $d$ -wave (long-dashed line) and interference between  $s$  and  $d$ -waves (dashed line).
- **Fig. 7:** Asymmetries for the reactions  $^{37}\vec{\text{K}}(\vec{e}, e'p)^{36}\text{Ar}(2_1^+)$  (top) and  $^{37}\vec{\text{Ar}}(\vec{e}, e'n)^{36}\text{Ar}(2_1^+)$  (bottom) calculated with the deformed HF orbital (solid line) and with the spherical orbital in PWIA (dashed line) and DWIA (short-dashed line). The scattering angle is  $\theta_e = 30$ . Other kinematical variables are as in Fig. 4.
- **Fig. 8:** Response functions for  $^{37}\vec{\text{K}}(\vec{e}, e'p)^{36}\text{Ar}$  in the spherical limit for different polarization directions and  $\phi_N$  values, calculated in PWIA and in DWIA.



Table 1: Coefficients  $\mathcal{F}^\lambda(\ell j; \ell' j')$  for the transition from the ground state ( $J_A = \frac{3}{2}, K_A = \frac{1}{2}$ ) in  $^{37}\text{Ar}$  ( $^{37}\text{K}$ ) to the first excited state ( $J_B^\pi = 2^+, K_B = 0$ ) in  $^{36}\text{Ar}$ .

	$\ell_j$ $\ell_{j'}$	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{7}{2}}$
$\lambda = 0$	$s_{\frac{1}{2}}$	0.080	0	0	0
	$d_{\frac{3}{2}}$	0	0.040	0	0
	$d_{\frac{5}{2}}$	0	0	0.011	0
	$g_{\frac{7}{2}}$	0	0	0	0.051
$\lambda = 2$	$s_{\frac{1}{2}}$	0	0.023	-0.028	0
	$d_{\frac{3}{2}}$	0.023	0.024	0.006	-0.029
	$d_{\frac{5}{2}}$	-0.028	0.006	0.001	-0.007
	$g_{\frac{7}{2}}$	0	-0.029	-0.007	0.037

Table 2: Coefficients  $\mathcal{G}_l^L(\ell j; \ell' j')$  for the same transition as in table I.

	$\ell_j$ $\ell'_{j'}$	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{7}{2}}$
$L = 0$	$s_{\frac{1}{2}}$	-0.048	0	0	0
	$d_{\frac{3}{2}}$	0	-0.048	0.016	0
	$d_{\frac{5}{2}}$	0	0.016	0.006	0
	$g_{\frac{7}{2}}$	0	0	0	-0.031
$L = 2$	$s_{\frac{1}{2}}$	0	-0.068	0.004	0
	$d_{\frac{3}{2}}$	-0.068	0.010	0.010	0.004
	$d_{\frac{5}{2}}$	0.004	0.010	$< 10^{-3}$	-0.018
	$g_{\frac{7}{2}}$	0	0.004	-0.018	0.047
$L = 4$	$s_{\frac{1}{2}}$	0	0	0	0.027
	$d_{\frac{3}{2}}$	0	-0.005	0.002	-0.014
	$d_{\frac{5}{2}}$	0	0.002	-0.001	0.005
	$g_{\frac{7}{2}}$	0.027	-0.014	0.005	-0.008

Table 3: Coefficients  $\mathcal{G}_t^L(\ell j; \ell' j')$  for the same transition as in table I.

	$\ell_j$ $\ell'_{j'}$	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{7}{2}}$
$L = 2$	$s_{\frac{1}{2}}$	0	-0.068	-0.003	0
	$d_{\frac{3}{2}}$	-0.068	0.009	0.006	-0.003
	$d_{\frac{5}{2}}$	-0.003	0.006	-0.005	-0.022
	$g_{\frac{7}{2}}$	0	-0.003	-0.022	0.042
$L = 4$	$s_{\frac{1}{2}}$	0	0	0	0.014
	$d_{\frac{3}{2}}$	0	-0.003	0.001	-0.007
	$d_{\frac{5}{2}}$	0	0.001	$< 10^{-3}$	0.002
	$g_{\frac{7}{2}}$	0.014	-0.007	0.002	-0.004

Table 4: Weights of the  $\ell j$ -projection of the deformed odd-nucleon wave function in  $^{37}\text{Ar}$  and  $^{37}\text{K}$ . Only values larger than  $10^{-3}$  have been tabulated.

$\ell_j$	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$d_{\frac{5}{2}}$	$g_{\frac{7}{2}}$	$g_{\frac{9}{2}}$
$^{37}\text{Ar}$	0.191	0.746	0.054	0.007	0.002
$^{37}\text{K}$	0.194	0.741	0.056	0.007	0.002

Table 5: Binding energies, charge r.m.s radii, ground state magnetic moments, intrinsic quadrupole moments, moments of inertia and decoupling parameters for  $^{37}\text{Ar}$  and  $^{37}\text{K}$ . Theoretical results obtained from DDHF calculations are compared to available experimental data<sup>[23]</sup>.

Nuclei	$B(\text{MeV})$		$r_c(\text{fm})$	$\mu_{3/2}$		$Q_0(\text{fm}^2)$	$\mathcal{I}(\text{MeV}^{-1})$	$a$
	exp.	th.	th.	exp.	th.	th.	th.	th.
$^{37}\text{Ar}$	-315.50	-312.28	3.41	$.95 \pm .20$	1.250	-43.09	1.89	-1.159
$^{37}\text{K}$	-308.57	-305.88	3.43	.203	.194	-32.39	1.87	-1.140